-
-
-
-

- $\mathbb{N} = \{1, 2, 3, \dots\}$
- $\mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \dots\}$
- $\mathbb{Q} = \left\{ \frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{Z} \setminus \{0\} \right\}$
- $\mathbb{R}$
- $\mathbb{C}$

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$$

$n\mathbb{Z} = \{0, \pm n, \pm 2n, \dots\} n$

$-5|10 a|b k a = bk \in \mathbb{Z} b a a, b$

$0 \leq r < |d| n = qd + r q, r d \neq 0, n \in \mathbb{Z}$

dn

n, m

$$(n, m) = \{d \in \mathbb{N} : d|n \wedge d|m\}$$

$(2, 5) = 1 (n, m) = 1 n, m (6, 10) = 2 (n, m)$

$$badd|bd|a$$

$$(n,m)=(m,r)n=qm+r$$

$$d \leq (m,r)d|r=n-qmn, mrd|md|nd=(m,r)d=(n,m)$$
$$d=(m,r)(m,r) \leq d(m,r)|n(m,r)|mm, rn(m,r)|n(m,r)|m(m,r)|r$$

$$(n,m)=(m,r)0 \leq r < mn=qm+r(n,m)=nm=00 \leq m < n$$

4753

$$(53,47)=[53=1\cdot 47+6]$$
$$(47,6)=[47=7\cdot 6+5]$$
$$(6,5)=1$$

$$(224,63)=[224=3\cdot 63+35]$$
$$(63,35)=[63=1\cdot 35+28]$$
$$(35,28)=[35=1\cdot 28+7]$$
$$(28,7)=[28=4\cdot 7+0]$$
$$(7,0)=7$$

$$a,b$$

$$(a,b)=_{u,v}\{au+bv\in\mathbb{N}\}$$

$$(a,b)=sa+tbs,t\in\mathbb{Z}$$

$$(a,b)\in a\mathbb{Z}+b\mathbb{Z}$$

$$s,t$$

$$(234,61)=[{\scriptstyle 234=3\cdot 61+51}\Rightarrow 51=234-3\cdot 61]$$
$$(61,51)=[{\scriptstyle 61=1\cdot 51+10}\Rightarrow 10=61-1\cdot 51=61-1\cdot(234-3\cdot 61)=-1\cdot 234+4\cdot 61]$$
$$(51,10)=[{\scriptstyle 51=5\cdot 10+1}\Rightarrow 1=51-5\cdot 10=51-5\cdot(-1\cdot 234+4\cdot 61)=6\cdot 234-23\cdot 61]$$
$$(10,1)=1$$

$$(234,61)=1=6\cdot 234-23\cdot 61$$

$$a|ca|bc(a,b)=1a,b,c$$

$$a|ca|\,(sac+tbc)a|tbca|sacc = sac+tbcc1 = sa+tbs, t$$

$$e|de|ne|med = (n,m)$$

$$(an,am) = a(n,m)$$

$$p|bp|ap|abp$$

$$dsn+tme|n,md = sn+tms, t$$

$$p|bpp|abpsab+tpb = bbsa+tp = 1s, t(p,a) = 1p \nmid a$$

$$n,m$$

$$(n,m) = \{d \in \mathbb{N} : n|d \wedge m|d\}$$

$$[2,5] = 10[6,10] = 30[n,m]$$

$$[n,m]\,|an|am|a$$

$$[6,4]\,(6,4) = 12 \cdot 2 = 24 = 6 \cdot 4[n,m]\,(n,m) = |nm|$$

$$[n,m]r \neq 0n,m|rn,m|\,[n,m]n,m|a0 \leq r < [n,m]a = q\,[n,m]+rq,r$$
$$[n,m]\,|aa = q\,[n,m]$$

$$n = \prod_{i=1}^{\infty} p_i^{\beta_i} = p_1^{\beta_1} p_2^{\beta_2} p_3^{\beta_3} \ldots \qquad m = \prod_{i=1}^{\infty} p_i^{\alpha_i} = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} \ldots$$

$$\alpha_i, \beta_i \geq 0$$

$$(n,m) = \prod_{i=1}^{\infty} p_i^{(\alpha_i,\beta_i)} \qquad [n,m] = \prod_{i=1}^{\infty} p_i^{(\alpha_i,\beta_i)}$$

$$[n,m]\,(n,m) = |nm|\alpha+\beta = (\alpha,\beta)+(\alpha,\beta)\alpha,\beta$$

$$ks_1n_1 + \cdots + s_kn_k = ds_1, \ldots, s_kn_1, \ldots, n_kd$$

$$nbaa \equiv b \ \ (\ n)a = b + knk \in \mathbb{Z}n|a - bna, b \in \mathbb{Z}n$$

$$a + c \equiv b + d \ \ (\ n)ac \equiv bd \ \ (\ n)a \equiv b, c \equiv d \ \ (\ n)nn$$

$$333^{333}$$

$$333^{333} = 3^{333} \cdot 111^{333}$$

$$111 \equiv 1 \quad (\ 10) \Rightarrow 111^{333} \equiv 1^{333} \equiv 1 \quad (\ 10)$$
$$3^{333} = 3^{4 \cdot 83 + 1} = \left(3^4\right)^{81} \cdot 3 = 81^{83} \cdot 3 \equiv 1^{83} \cdot 3 \quad (\ 10)$$
$$333^{333} = 3^{333} \cdot 111^{333} \equiv 3 \quad (\ 10)$$

3

$$61x \equiv 1 \ \ (\ 234)0 \leq x \in \mathbb{Z}$$

$$x \equiv -231 = 6 \cdot 234 - 23 \cdot 61k, x(234, 61) = 123461161x + 234k \equiv 1k \in \mathbb{Z}$$
$$x = 211x(\ 234)$$

$$x \equiv b \ \ (\ m)x \equiv a \ \ (\ n)nmxa, b \in \mathbb{Z}n, m$$

$$bsn + atmxsn + tm = 1s, t \in \mathbb{Z}(n, m) = 1$$

$$bsn + atm \equiv atm \equiv a \cdot 1 \equiv a \quad (\ n)$$
$$bsn + atm \equiv bsn \equiv b \cdot 1 \equiv b \quad (\ m)$$

$$k \in \mathbb{Z}x' = x + kmnx = bsn + atm$$
$$nmxnmnm(a, b)nmnmx(a, b)nmx$$

$$n = 5, m = 3 {-} 1 \cdot 5 + 2 \cdot 3 = 1(5, 3) = 1x \equiv 2 \ \ (\ 5)x \equiv 1 \ \ (\ 3)x \in \mathbb{Z}$$
$$7 \equiv 2 \ \ (\ 5)7 \equiv 1 \ \ (\ 3)x = 1 \cdot (-5) + 2 \cdot 6 = 7s = -1, t = 2$$

$$mx\{a_i \, (m_i) : 1 \leq i \leq k\}m\{m_1, \ldots, m_k\}$$

$$\begin{cases} x \equiv a_1 \quad (\ m_1) \\ \vdots \\ x \equiv a_k \quad (\ m_k) \end{cases}$$

$$15 \equiv 0 \ \ (\ 5)15 \equiv 0 \ \ (\ 3)3 \cdot 5 = 15y = 7y \equiv 3 \ \ (\ 7)y \equiv 2 \ \ (\ 5)y \equiv 1 \ \ (\ 3)y \in \mathbb{Z}$$
$$y = 52(15, 7) = 1y \equiv 7 \ \ (\ 15)y \equiv 2 \ \ (\ 5)y \equiv 1 \ \ (\ 3)$$